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BULK VISCOUS STRING COSMOLOGICAL MODEL WITH COSMOLOGICAL TERM Λ IN FRW SPACE-TIME

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Abstract: We have investigated bulk viscous string cosmological model with cosmological term Λ in FRW space-time. To obtain the deterministic solution we assume that bulk viscosity ξ is inversely proportional to the expansion θ; proper energy density ρ is proportional to the string tension density λ and Λ is proportional to R^{-2}, where R is a scale factor. Various physical and geometrical aspects of the model are also discussed.

Keywords: FRW space-time, viscous fluid, cosmological term.

1. Introduction

Cosmic strings play a significant role during the early stage of evolution of Universe (Kibble[10]) and gives rise to density perturbations which lead to the formation of galaxies (Zel'dovich [21]). These strings have stress energy and couple to gravitational field. Therefore, it is interesting to study the gravitational effect which arises from string. The general relativistic treatment of strings was initiated by (Letelier [11]). Stachel [16] considered a massless (geometric string) to develop a realistic treatment of strings consequently many Relativists viz. Banerjee et al. [5], Tikekar and Patel [17], Bali and Anjali [3], Wang [19] to mention a few, obtained string cosmological models under different space-times.

The presence of viscosity in the fluid content, has been found very useful to explain many physical phenomena in the study of homogeneous cosmological models and different picture of universe appeared at the initial stage of evolution of universe due to dissipative process caused by viscosity as viscosity counteracts the cosmological collapse. The effect of viscosity in the evolution of cosmological models is investigated by Misner [12]. A number of researchers viz. Bali [2], Tiwari and Sharma [18], Sharma & Tyagi [15], Dubey et al. [9], Parikh et al. [13] investigated viscous fluid cosmological models in different contexts.

The cosmological constant (Λ) has played a stimulating role in gravitation theory. Also cosmological constant is an outstanding problem in cosmology. Many researchers have taken keen interests to resolve this problem. Zel'dovich[20] in his most innovative way revived the issue of cosmological constant by identifying it with vacuum energy density due to quantum fluctuations. The cosmological models with time dependent cosmological term have been studied by Berman[7], Chen and Wu[8], Beesham[6], Bali and Singh[4], Samdurkar[14], to name a few. Recently Bali [1] investigated cosmological models for radiation dominated phase with vacuum energy density in Friedman-Robertson-Walker (FRW) model.

Motivated by the investigations discussed above, we have investigated bulk viscous string cosmological model with cosmological term Λ in FRW space-time. To obtain the deterministic solution of the model we have assumed that bulk viscosity ξ is inversely proportional to the expansion θ; proper energy density ρ is proportional to the string tension density λ and Λ is proportional to R^{-2}, where R is a scale factor. Various physical and geometrical aspects of the model are also discussed.

2. Metric and Field Equations

We consider FRW metric in the form of

$$ds^2 = -dt^2 + R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 sin^2\theta d\varphi^2\right] \quad (1)$$

where the parameter k takes values as 1, 0 or -1.

The Einstein's field equation (in the gravitational unit c=8πG=1) is given by

$$R_i^j - \frac{1}{2}Rg_i^j + \Lambda g_i^j = -T_i^j \quad (2)$$

where R_i^j is Ricci tensor, R=$g^{ij} R_{ij}$ is scalar curvature.

The energy momentum tensor T_i^j for a cloud of strings with viscous fluid is defined as

$$T_i^j = \rho v_i v^j - \lambda x_i x^j - \xi\theta\left(v_i v^j + g_i^j\right) \quad (3)$$

where $v_i v^i = -x_i x^i = -1$ and $v^i x_i = 0$ (4)

Here ρ is proper energy density, λ is string tension density, x^i is unit space like vector specifying the direction of strings and v^i is unit time like vector satisfying the condition

$$g^{ij} v_i v_j = -1 \quad (5)$$

In a co-moving coordinate system, we have $v^i = (0,0,0,1)$.

The energy momentum tensor for the metric (1) using equation (3) is given by

$$T_1^1 = T_2^2 = T_3^3 = -(\lambda + \xi\theta); T_4^4 = \rho \quad (6)$$

The Einstein's field equation (2) for FRW metric (1) together with equation (6) leads to the following system of equations:

$$\frac{2R_{44}}{R} + \frac{R_4^2}{R^2} + \frac{K}{R^2} + \Lambda = \lambda + \xi\theta \quad (7)$$

$$\frac{3R_4^2}{R^2} + \frac{3K}{R^2} + \Lambda = \rho \tag{8}$$

3. Solution of the Field Equations

The field equations (7) and (8) are two equations with 4 unknown parameters. So to obtain the explicit solution we assume that proper energy density ρ is proportional to the string tension density λ,

i.e. $\rho = \alpha\lambda$ (9)

and bulk viscosity ξ is inversely proportional to the expansion θ,

i.e. $\xi\theta = l$ (10)

where α and l are constant of proportionality and $\alpha > 3$, $l \neq 0$.

Using the above conditions in equations (7) and (8) we have

$$\frac{2\alpha R_{44}}{R} + \frac{(\alpha-3)R_4^2}{R^2} + \frac{(\alpha-3)K}{R^2} + (\alpha-1)\Lambda - l\alpha = 0 \tag{11}$$

To get the deterministic solution, we also assume that Λ is proportional to R^{-2} as mentioned by Chen & Wu [8]

$$\Lambda = \frac{\beta}{R^2} \tag{12}$$

where β is a constant.

Using equation (12) in equation (11) we get

$$\frac{2\alpha R_{44}}{R} + \frac{(\alpha-3)R_4^2}{R^2} = l\alpha - \left[\frac{(\alpha-3)K+(\alpha-1)\beta}{R^2}\right] \tag{13}$$

Equation (13) leads to

$$2R_{44} + \frac{(\alpha-3)R_4^2}{\alpha R} = lR - \left[\frac{(\alpha-3)K+(\alpha-1)\beta}{\alpha R}\right] \tag{14}$$

Now let $R_4 = f(R)$ then equation (14) becomes

$$\frac{df^2}{dR} + \frac{(\alpha-3)f^2}{\alpha R} = lR - \left[\frac{(\alpha-3)K+(\alpha-1)\beta}{\alpha R}\right] \tag{15}$$

On integrating equation (15), we get

$$f^2 = \frac{l\alpha R^2}{3(\alpha-1)} - \left[\frac{(\alpha-3)K+(\alpha-1)\beta}{\alpha-3}\right] + \frac{M}{R^{\frac{\alpha-3}{\alpha}}} \tag{16}$$

where M is constant of integration.

From equation (16) we get

$$\int \frac{dR}{\sqrt{\frac{l\alpha R^2}{3(\alpha-1)} - \left[\frac{(\alpha-3)K+(\alpha-1)\beta}{\alpha-3}\right] + \frac{M}{R^{\frac{\alpha-3}{\alpha}}}}} = \int dt + N = t + N \tag{17}$$

where N is constant.

The value of R can be determined by the equation (17).

Hence, by suitable transformation, metric (1) reduces to the form

$$ds^2 = -\frac{dT^2}{\left[\frac{l\alpha T^2}{3(\alpha-1)} - \left\{\frac{(\alpha-3)K+(\alpha-1)\beta}{\alpha-3}\right\} + \frac{M}{T^{\frac{\alpha-3}{\alpha}}}\right]} + T^2\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 sin^2\theta d\varphi^2\right] \quad (18)$$

where R=T.

4. Special Models

I. Model when $M = 0$

To obtain the deterministic solution in terms of cosmic time 't', we put $M = 0$ in equation (16) which leads to

$$f^2 = \frac{l\alpha R^2}{3(\alpha-1)} - \left[\frac{(\alpha-3)K+(\alpha-1)\beta}{\alpha-3}\right] \quad (19)$$

Equation (19) can be rewrite as

$$\left(\frac{dR}{dt}\right)^2 = uR^2 - v \quad (20)$$

where $u = \frac{l\alpha}{3(\alpha-1)}$ and $v = \left[\frac{(\alpha-3)K+(\alpha-1)\beta}{\alpha-3}\right]$

Equation (20) leads to

$$\int \frac{dR}{\sqrt{uR^2-v}} = \int dt \quad (21)$$

On integrating equation (21), we get

$$R = \sqrt{\frac{v}{u}}\cosh\left(\sqrt{u}t + L\right) \quad (22)$$

where L is integrating constant.

Therefore metric (1) reduces to the form

$$ds^2 = -dt^2 + \frac{v}{u}\cosh^2\left(\sqrt{u}t + L\right)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 sin^2\theta d\varphi^2\right] \quad (23)$$

II. Model in the Absence of Bulk Viscosity

The space-time in the absence of bulk viscous fluid is as follows

$$ds^2 = -\frac{dT^2}{\left[-\left\{\frac{(\alpha-3)K+(\alpha-1)\beta}{\alpha-3}\right\} + \frac{M}{T^{\frac{\alpha-3}{\alpha}}}\right]} + T^2\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 sin^2\theta d\varphi^2\right] \quad (24)$$

5. Some Physical and Geometrical Features

Proper energy density (ρ) for the model (18) is given by

$$\rho = \frac{l\alpha}{\alpha-1} - \left(\frac{2\alpha\beta}{\alpha-3}\right)\frac{1}{T^2} + \frac{3M}{T^{\frac{3(\alpha-1)}{\alpha}}} \tag{25}$$

String tension density ($\boldsymbol{\lambda}$) for the model (18) is given by

$$\lambda = \frac{l}{(\alpha-1)} - \left[\frac{2\beta}{(\alpha-3)}\right]\frac{1}{T^2} + \frac{3M}{\alpha T^{\frac{3(\alpha-1)}{\alpha}}} \tag{26}$$

Spatial volume (R^3) for the model (18) is given by

$$R^3 = T^3 \tag{27}$$

Expansion θ for the model (18) is given by

$$\theta = 3\sqrt{\frac{l\alpha}{3(\alpha-1)} - \left[\frac{(\alpha-3)K+(\alpha-1)\beta}{\alpha-3}\right]\frac{1}{T^2} + \frac{M}{T^{3\left(\frac{\alpha-1}{\alpha}\right)}}} \tag{28}$$

Cosmological term Λ for the model (18) is given by

$$\Lambda = \frac{\beta}{T^2} \tag{29}$$

Proper energy density (ρ), String tension density (λ), spatial volume (R^3), Expansion θ and Cosmological term Λ for the model (23) are given by

$$\rho = 3u + \frac{[3(K-v)+\beta]v\,\mathrm{sech}^2(\sqrt{u}t+L)}{v} \tag{30}$$

$$\lambda = \frac{3u}{\alpha} + \frac{[3(K-v)+\beta]v\,\mathrm{sech}^2(\sqrt{u}t+L)}{v\alpha} \tag{31}$$

$$R^3 = \left(\frac{v}{u}\right)^{\frac{3}{2}}\cosh^3(\sqrt{u}t+L) \tag{32}$$

$$\theta = 3\sqrt{u} \tag{33}$$

$$\Lambda = \frac{\beta u\,\mathrm{sech}^2(\sqrt{u}t+L)}{v} \tag{34}$$

Proper energy density (ρ), String tension density (λ) and Expansion θ for the model (24) are given by

$$\rho = -\left(\frac{2\alpha\beta}{\alpha-3}\right)\frac{1}{T^2} + \frac{3M}{T^{\frac{3(\alpha-1)}{\alpha}}} \tag{35}$$

$$\lambda = -\left[\frac{2\beta}{(\alpha-3)}\right]\frac{1}{T^2} + \frac{3M}{\alpha T^{\frac{3(\alpha-1)}{\alpha}}} \tag{36}$$

$$\theta = 3\sqrt{-\left[\frac{(\alpha-3)K+(\alpha-1)\beta}{\alpha-3}\right]\frac{1}{T^2} + \frac{M}{T^{3\left(\frac{\alpha-1}{\alpha}\right)}}} \tag{37}$$

6. Conclusion

In the model (18), we observe that for the proper energy density [equation (25)], the reality condition $\rho \geq 0$ leads to

$$\rho = \frac{l\alpha}{\alpha-1} - \left(\frac{2\alpha\beta}{\alpha-3}\right)\frac{1}{T^2} + \frac{3M}{T^{\frac{3(\alpha-1)}{\alpha}}} \geq 0$$

which does not vanish at T→∞ and $\alpha > 1$ and $l > 0$. In the presence of viscous fluid, it becomes

$$\rho = \frac{l\alpha}{\alpha-1}.$$

Thus α ≠ 1. For α = 1, the model leads to string dust model.

Also from (26), we can observe that string tension density (λ) does not vanish at T→∞. We also observed that the model starts with big–bang at T = 0 and expansion θ decreases as time T increases and it becomes constant at T→∞. From (29) it is also observed that cosmological term Λ is decreasing as T is increasing T and at late time it approaches to zero which is supported by current observations.

Also, for the model (23), we can observe that proper energy density (ρ) and string tension density (λ) do not vanish at t→∞ and expansion θ becomes zero as t→∞ and u→0. Also cosmological term Λ, is decreasing function of cosmic time t for particular model.

The model (24) also starts with big–bang at T = 0. For this model, we can observe that proper energy density (ρ), string tension density (λ) and expansion θ decrease as time T increase and vanish at T→∞.

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